Mathematics

Class XII

Sample Paper – 10

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. Find the value of x, if

$$\begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix} + \begin{bmatrix} 3 & x \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 10 \\ 8 & 6 \end{bmatrix}$$

2. Differentiate $\cos\sqrt{x}$ w.r.t. x

3. Is the differential equation given by $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y\sin y = 0$, linear or nonlinear. Give reason.

4. Find the angle between following pairs of line

$$\frac{5-x}{-2} = \frac{y+3}{1} = \frac{1-z}{3} \text{ and } \frac{x}{3} = \frac{1-y}{-2} = \frac{z+5}{-1}$$

OR

Find the angle between following pairs of line

$$\frac{-x+2}{-2} = \frac{y-1}{7}, \frac{z+3}{-3} \text{ and } \frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$$

SECTION – B

5. For the power set of all subsets of a non-empty set, a relation ARB is defined if and only if $A \subset B$. Is R an equivalence relation on the Power set?

6. Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and

$C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

7. Find :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$

8.

Find

$$\int \frac{dx}{5 - 8x - x^2}$$

OR

Evaluate: $\int \frac{x^2+1}{x^4+1} dx.$

9. Form the differential equation of parabolas having vertex at the origin and axis along positive y axis

10. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=15$ then find $|\vec{x}|$.

OR

Find the distance between the parallel planes

$\vec{r}.\ 2i-1\hat{j}+3\hat{k}\ =4$ and $\vec{r}.\ 6\hat{i}-3\hat{j}+9\hat{k}\ +13=0$

11. A company has two plants to manufacturing scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30%. At plant I, 30% of the scooters are rated of standard quality and at plant II, 90% of the scooters are rated of standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it is manufactured by plant II.

12. In a game Rohan wins a rupee for a six and loses a rupee for any other number. Rohan has been asked to throw the dice thrice but he has to quit when he get six. Find his expected earnings.

OR

How many times must a fair coin be tossed so that the probability of getting atleast one head is more than 90%.

SECTION – C

13. A relation R on the set of complex numbers is defined by

$$z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2}$$

Show that R is an equivalence relation.

OR

Let f: N $\rightarrow$ N be defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2} & \text{if n is odd} \\ \dfrac{n}{2} & \text{if n is even} \end{cases}$$

Find whether the function f is bijective or not.

14. Prove the following equation.

$$2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

15. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

16. Differentiate $\tan^{-1}\left\{\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ w.r.t. $\cos^{-1} x^2$

OR

If $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$, find $\dfrac{dy}{dx}$

17. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}x$, $x\neq0$

18. Find the equation of a tangent to the curve given by $x=a\sin^3 t$, $y=b\cos^3 t$ at a point, where $t=\frac{\pi}{2}$.

19. Find

$$\int\frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta$$

20. Evaluate: $\int_0^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

21. Solve: $xdy-ydx=\sqrt{x^2+y^2}dx$

OR

Solve: $2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$

22. Find the shortest distance between the lines $\vec{r_1}$ and $\vec{r_2}$ whose vector equations are

$$\vec{r}_1=\hat{i}+\hat{j}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$$

$$\vec{r}_2=2\hat{i}+\hat{j}-\hat{k}+\mu\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$$

23. Find the equation of the plane determined by the point A(3, - 1, 2), B(5, 2, 4) and C(-1, -1, 6) and hence find the distance between the plane and the point P(6, 5, 9).

SECTION – D

24. Obtain the inverse of following matrix using elementary operations

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

OR

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that A is a root of polynomials $f(x) = x^3 - 23x - 40$

25. Find the points of maxima and minima of the function f given by $f(x) = (x - 2)^4 (x+1)^3$

26. Find the area of the smaller region bounded by the curves $x^2 + y^2 = 4$ and $y^2 = 3(2x - 1)$.

OR

Find the area of the region bounded by lines $y = \frac{5}{2}x - 5; x + y - 9 = 0; y = \frac{3}{4}x - \frac{3}{2}$

27. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find the distance of the plane obtained above, from the origin.

OR

Find the distance of the point (2, 12, 5) from the point of intersection of the line

$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$.

28. A co-operative society of farmers has 50 hectares of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs. 10,500 and Rs. 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of Rs. 20 lakh and Rs. 10 lakh per hectare. Further, no more than Rs. 800 lakh of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society?

29. Two dice are rolled twice. Find the probability distribution of the random variable X, which denotes the number of doublets. Find its mean and variance.

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Sample Paper – 10 Solution

SECTION – A

1. By observation we find that

2 + x = 10

x = 8.

2. $\frac{d}{dx}\left(\cos\sqrt{x}\right)$

$=-\left(\sin\sqrt{x}\right)\frac{d}{dx}\left(\sqrt{x}\right)$

$\frac{dy}{dx}=\frac{-\sin\sqrt{x}}{2\sqrt{x}}$

3. DE:

$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}+\frac{dy}{dx}+y\sin y=0$

It is linear, since $y\times\sin y$ is product of two different functions, and their individual power is one.

4. Let θ be the angles between, the given two lines

So, the angle between them given their direction cosines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

substituting we get

$a_1 = 2$

$a_2 = 3$

$b_1 = 1$

$b_2 = 2$

$c_1 = -3$

$c_2 = -1$

$$\theta = \cos^{1}\left(\frac{11}{14}\right)$$

OR

Let θ be the angles between, the given two lines

So, the angle between them given their direction cosines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

substituting we get

$$a_1a_2 + b_1b_2 + c_1c_2 = (2)(-1) + (7)(2) + (-3)(4) = 0$$

$$\theta = \cos^{1}(0) = \frac{\pi}{2}$$

SECTION – B

5. Let X be the non-empty set for which P(X) is the power set.

$ARB \Leftrightarrow A \subset B$

i. $ARA \Leftrightarrow A \subset A$, every set is a subset of itself. R is reflexive

ii. If $A, B, C \in P(X)$

$ARB \Leftrightarrow A \subset B$, $BRC \Leftrightarrow B \subset C$

$A \subset B$ and $B \subset C \Rightarrow A \subset C$

So ARC; Hence R is transitive.

iii. $ARB \Leftrightarrow A \subset B$ does not imply $B \subset A$

So $B \not R A$

R is not symmetric

R is reflexive, transitive but not symmetric $\Rightarrow$R is not an equivalence relation

6. We have,

$2A - 3B + 5C = O$

$2A = 3B - 5C$

$$2A = 3\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$2A = \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} - \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix}$$

$$2A = \begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$A = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$

7.

$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$

$$\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$$

$$= 2\int \frac{-(\cos^2 x - \sin^2 x)}{2\sin x \cos x} dx$$

$$= -2\int \frac{\cos 2x}{\sin 2x} dx$$

$$= -\int \frac{2\cos 2x}{\sin 2x} dx$$

$$= -\log|\sin 2x| + C \quad \text{.......}(\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c)$$

8.

$$\int \frac{dx}{5 - 8x - x^2}$$

$$= \int \frac{dx}{-(x^2 + 8x - 5)}$$

$$= \int \frac{dx}{-(x^2 + 8x + 16 - 16 - 5)}$$

$$= \int \frac{dx}{-[(x+4)^2 - 21]}$$

$$= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$$

$$= \frac{1}{2\sqrt{21}} \log\left|\frac{\sqrt{21} + (x+4)}{(x+4) - \sqrt{21}}\right| + C$$

$$= \frac{1}{2\sqrt{21}} \log\left|\frac{x + 4 + \sqrt{21}}{x + 4 - \sqrt{21}}\right| + C$$

OR

Let $I = \int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$

(Dividing numerator and denominator by x^2)

$$I = \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}\,dx$$

Substituting $x-\frac{1}{x} = t, \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$ we get,

$$I = \int \frac{dt}{t^2+2} = \int \frac{dt}{t^2+\left(\sqrt{2}\right)^2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{x-\frac{1}{x}}{\sqrt{2}}\right]$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{x^2-1}{\sqrt{2}x}\right]+c$$

9. Equation of parabola having vertex at the origin and axis along positive y axis is given by

$x^2 = 4ay$, where a is a parameter

Differentiating w.r.t. x

$$\frac{d}{dx}x^2 = 4a\frac{d}{dx}y$$

$$2x = 4a\frac{dy}{dx}$$

$$a = \frac{x}{2\frac{dy}{dx}}$$

substituting value of a in equation $x^2 = 4ay$

$$x^2 = 4 \times \frac{x}{2 \times \frac{dy}{dx}} \times y$$

$$x\frac{dy}{dx} = 2y$$

which is the required differential equation

10. $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 15$

$$\Rightarrow \vec{x}.\vec{x} - \vec{a}.\vec{a} = 15$$

$$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 15$$

$$\Rightarrow |\vec{x}|^2 - 1 = 15$$

$$\Rightarrow |\vec{x}|^2 = 16$$

$$\Rightarrow |\vec{x}| = \pm 4$$

OR

Distance between the parallel planes

is given by $\dfrac{|d-k|}{|\vec{n}|}$

$\vec{r}.\ 6\hat{i}-3\hat{j}+9\hat{k}\ +13=0$

$\Rightarrow \vec{r}.\ 2\hat{i}-\hat{j}+3\hat{k}\ =-\dfrac{13}{3}$

$\vec{r}.\ 2i-1\hat{j}+3\hat{k}\ =4$ and $\vec{r}.\ 2\hat{i}-\hat{j}+3\hat{k}\ =-\dfrac{13}{3}$

$\therefore$ the distance between the given parallel planes

is $\dfrac{\left|4-\left(-\dfrac{13}{3}\right)\right|}{\sqrt{2\times 2+(-1\times -1)+3\times 3}}$

$=\dfrac{\left|4+\left(\dfrac{13}{3}\right)\right|}{\sqrt{4+1+9}}=\dfrac{\dfrac{25}{3}}{\sqrt{14}}=\dfrac{25}{3\sqrt{14}}$

11. $P(I)=\dfrac{70}{100};\ P(II)=\dfrac{30}{100};$

E : standard quality;

$P(E/I)=\dfrac{30}{100};\ P(E/II)=\dfrac{90}{100}$

$P(II/E)=\dfrac{P(II).P(E/II)}{P(I).P(E/I)+P(II).P(E/II)}$

$=\dfrac{\dfrac{30}{100}\times\dfrac{90}{100}}{\dfrac{70}{100}\times\dfrac{30}{100}+\dfrac{30}{100}\times\dfrac{90}{100}}=\dfrac{9}{16}$

12. Probability of getting a six = $\frac{1}{6}$

Probability of getting any other number = $\frac{5}{6}$

Following are the proceedings of game

1. got a six, hence game over
2. got some other number, then six appears
3. got six in 3^{rd} chance
4. did not get six at all

Expected earnings

$$=\frac{1}{6}\times 1+\frac{5}{6}\times(-1)\times\frac{1}{6}(1)+\frac{5}{6}\times(-1)\times\frac{5}{6}\times(-1)\times\frac{5}{6}(-1)+\frac{5}{6}\times(-1)\times\frac{5}{6}\times(-1)\times\frac{1}{6}(1)$$
$$=\frac{1}{6}+\frac{25}{216}-\frac{5}{36}-\frac{125}{216}=\frac{36}{216}+\frac{25}{216}-\frac{30}{216}-\frac{125}{216}=\frac{-94}{216}$$

OR

Let the coin be tossed n times,

Probability of getting a head = $\frac{1}{2}$

Probability of getting no head in n trials is = $\left(\frac{1}{2}\right)^n$

$\therefore$ Probability of getting atleast one head = 1- $\left(\frac{1}{2}\right)^n$ ...(1)

Probability of getting at least one head must be > 90% = 0.9 (given) ...(2)

From (1) and (2)

$$1-\left(\frac{1}{2}\right)^n > 0.9$$

$$\Rightarrow \left(\frac{1}{2}\right)^n < 1 - 0.9$$

$$\Rightarrow \left(\frac{1}{2}\right)^n < 0.1 \qquad \Rightarrow n \geq 4$$

SECTION – C

13. $z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2}$ is real

(i) R is reflexive : $z_1 R z_1 = \frac{z_1 - z_1}{z_1 + z_1} = 0, 0 \in$ real numbers

(ii) R is symmetric: $z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2} \in$ real numbers, $\Rightarrow \frac{z_2 - z_1}{z_2 + z_1} \in$ real numbers

$\Rightarrow z_2 R\ z_1$

(iii) R is transitive: Let $z_1 = a_1 + ib_1, z_2 = a_2 + ib_2, z_3 = a_3 + ib_3 \in C$,

such that

$z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2} \in$ real numbers and $z_2 R z_3 = \frac{z_2 - z_3}{z_2 + z_3} \in$ real numbers

$z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2} \in$ real numbers

$\Rightarrow \frac{(a_1 + ib_1) - (a_2 + ib_2)}{(a_1 + ib_1) + (a_2 + ib_2)} \in$ real numbers $\Rightarrow \frac{(a_1 - a_2) + i(b_1 - b_2)}{(a_1 + a_2) + i(b_1 + b_2)} \in$ real numbers

$\Rightarrow \left[\frac{(a_1 - a_2) + i(b_1 - b_2)}{(a_1 + a_2) + i(b_1 + b_2)} \times \frac{(a_1 + a_2) + i(b_1 + b_2)}{(a_1 + a_2) + i(b_1 + b_2)}\right] \in$ real numbers

$\Rightarrow$ The imaginary part $= \left[(a_1 - a_2) \times (b_1 + b_2) + (b_1 - b_2) \times (a_1 + a_2)\right] = 0$

$\Rightarrow 2[a_1 b_2 - a_2 b_1] = 0 \Rightarrow [a_1 b_2 - a_2 b_1] \Rightarrow a_1 b_2 = a_2 b_1$...(i)

Similary, $z_2 R z_3 = \frac{z_2 - z_3}{z_2 + z_3} \in R \Rightarrow a_2 b_3 = a_3 b_2$....(ii)

Multiplying, (i) and (ii),

$a_1 b_2 a_2 b_3 = a_2 b_1 a_3 b_2$

Case I : When $b_2 a_2 \neq 0$

$a_1 b_2 a_2 b_3 = a_2 b_1 a_3 b_2 \Rightarrow a_1 b_3 = b_1 a_3 \Rightarrow z_2 \in R$

Case II : When $b_2 a_2 = 0 \Rightarrow z_2 = 0 \in R$

$\Rightarrow$ R is transitive

The given relation R is (i) Reflexive (ii) Symmetric (iii) Transitive

$\Rightarrow$ The given relation R is an Equivalence relation

OR

$f: N \rightarrow N$

$$f(n)=\begin{cases}\dfrac{n+1}{2} & \text{if } n \text{ is odd}\\ \dfrac{n}{2} & \text{if } n \text{ is even}\end{cases}$$

Let $f(n_1)=f(n_2)$

Case 1: n_1, n_2 are odd

Let $f(n_1)=f(n_2)$

$\Rightarrow \dfrac{n_1+1}{2}=\dfrac{n_2+1}{2}$

$\Rightarrow n_1=n_2$

Case 2: n_1, n_2 are even

$f(n_1)=f(n_2) \Rightarrow \dfrac{n_1}{2}=\dfrac{n_2}{2} \Rightarrow n_1=n_2$

Case3: n_1 is odd and n_2 is even

$f(n_1)=f(n_2) \Rightarrow \dfrac{n_1+1}{2}=\dfrac{n_2}{2}$

$\Rightarrow n_1+1=n_2$

$\Rightarrow n_1 \neq n_2$

Hence,

$f(n_1)=f(n_2)$ does not imply $n_1=n_2 \ \forall\ n_1, n_2 \in N$

$\therefore f$ is not one-one

Function f is onto and hence, f is surjective.

So f is not bijective.

14. L.H.S. $= 2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8}$

$$= 2\left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right) + \sec^{-1}\frac{5\sqrt{2}}{7}$$

$$= 2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\times\frac{1}{8}}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{13}{39}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)}\right) + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right) = \tan^{-1}\left(\frac{\frac{25}{28}}{\frac{25}{28}}\right) = \tan^{-1}1$$

$$= \frac{\pi}{4}$$

15. Consider $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

$R_1 \rightarrow R_1 + (R_2 + R_3)$

$$\Rightarrow \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$\Rightarrow (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$C_1 \rightarrow C_1 - C_3; C_2 \rightarrow C_2 - C_3$

$$\Rightarrow (5x+4)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 4-x & 2x \\ x-4 & x-4 & x+4 \end{vmatrix}$$

$$=(5x+4)(4-x)^2\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 2x \\ -1 & -1 & x+4 \end{vmatrix}$$

$$=(5x+4)(4-x)^2$$

16. $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ w.r.t. $\cos^{-1}x^2$

Let

$$u=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$$

and $v=\cos^{-1}x^2$

$$u=\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$$

putting $x^2=\cos\theta$

$$=\tan^{-1}\left\{\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right\}$$

$$u=\tan^{-1}\left\{\frac{\sqrt{2\cos^2\frac{\theta}{2}}-\sqrt{2\sin^2\frac{\theta}{2}}}{\sqrt{2\cos^2\frac{\theta}{2}}+\sqrt{2\sin^2\frac{\theta}{2}}}\right\}$$

$$u=\tan^{-1}\left\{\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right\}$$

$$u=\tan^{-1}\left\{\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right\}$$

$$u=\tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right\}$$

$$u=\frac{\pi}{4}-\frac{\theta}{2}=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$$

$$\frac{du}{dx}=\frac{x}{\sqrt{1-x^4}}$$

Now,

$$v=\cos^{-1}x^2$$

$$\frac{dv}{dx}=\frac{-2x}{\sqrt{1-x^4}},\ \frac{du}{dv}=\frac{\frac{x}{\sqrt{1-x^4}}}{\frac{-2x}{\sqrt{1-x^4}}}=-\frac{1}{2}$$

OR

we have,

$$x=\frac{\sin^3 t}{\sqrt{\cos 2t}} \text{ and } y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$$

so,

$$x=\sin^3 t(\cos 2t)^{\frac{-1}{2}} \text{ and } y=\cos^3 t(\cos 2t)^{\frac{-1}{2}}$$

differentiating w.r.t. t

$$\frac{dx}{dt}=\frac{3\sin^2 t\cos t}{\sqrt{\cos 2t}}+\sin^3 t\times\frac{-1}{2}\times(\cos 2t)^{\frac{-3}{2}}\frac{d}{dx}(\cos 2t)$$

$$=\frac{3\sin^2 t.\cos t.\cos 2t+\sin^3 t.\sin 2t}{(\cos 2t)^{\frac{3}{2}}}$$

$$=\frac{3\sin^2 t.\cos t.(1-2\sin^2 t)+\sin^3 t.(2\sin t.\cos t)}{(\cos 2t)^{\frac{3}{2}}}$$

$$=\frac{3\sin^2 t.\cos t-4\sin^4 t.\cos t}{(\cos 2t)^{\frac{3}{2}}}$$

$$=\frac{\sin 2t.\sin 3t}{2(\cos 2t)^{\frac{3}{2}}}$$

$$\frac{dy}{dt}=\frac{-3\cos^2 t\sin t}{\sqrt{\cos 2t}}+\cos 3t\times\frac{-1}{2}\times(\cos 2t)^{\frac{-3}{2}}\frac{d}{dx}(\cos 2t)$$

$$=\frac{-3\cos^2 t.\sin t.\cos 2t+\cos^3 t.\sin 2t}{(\cos 2t)^{\frac{3}{2}}}$$

$$=\frac{3\cos^2 t.\sin t-4\cos^4 t.\sin t}{(\cos 2t)^{\frac{3}{2}}}$$

$$=\frac{-\sin 2t.\cos 3t}{(\cos 2t)^{\frac{3}{2}}}$$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\frac{-\sin 2t.\cos 3t}{(\cos 2t)^{\frac{3}{2}}}}{\frac{\sin 2t.\sin 3t}{2(\cos 2t)^{\frac{3}{2}}}}=-\cos 3t$$

17. Let

$$u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) \text{ and } v = \tan^{-1} x$$

putting $x = \tan\theta$

$$u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$u = \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$$

differentiating w.r.t.x

$$\frac{du}{dx} = \frac{1}{2\left(1+x^2\right)}$$

$$\frac{dv}{dx} = \frac{1}{1+x^2}$$

$$\frac{du}{dv} = \frac{\frac{1}{2\left(1+x^2\right)}}{\frac{1}{1+x^2}} = \frac{1}{2}$$

18. Here, $x = a\sin^3 t$, $y = b\cos^3 t$ ....(1)

Differentiating (1) w.r.t. t

$$\frac{dx}{dt} = 3a\sin^2 t \times \cos t \text{ and}$$

$$\frac{dy}{dt} = -3b\cos^2 t \times \sin t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3b\cos^2 t \times \sin t}{3a\sin^2 t \times \cos t} = -\frac{b}{a}\cot t$$

$\therefore$ Slope of the tangent at $t = \frac{\pi}{2}$

$$\left.\frac{dy}{dx}\right]_{\frac{\pi}{2}} = -\frac{b}{a}\cot\frac{\pi}{2} = 0$$

Hence, equation of tangent is given by

$$y - b\cos^3\frac{\pi}{2} = 0 \text{ or } y = 0$$

19.

Given that

$$I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$$

Put, $\sin\theta = t$

$\Rightarrow \cos\theta d\theta = dt$

$$I = \int \frac{\cos\theta}{(4+\sin^2\theta)\left[5-4(1-\sin^2\theta)\right]} d\theta$$

$$I = \int \frac{dt}{(4+t^2)\left[5-4(1-t^2)\right]} I = \int \frac{dt}{\left(4+t^2\right)\left(5-4+4t^2\right)}$$

$$I = \int \frac{dt}{(4+t^2)(1+4t^2)}$$

Using partial fraction,

$$\frac{1}{(4+t^2)(1+4t^2)} = \frac{A}{4+t^2} + \frac{B}{1+4t^2} \quad ......(i)$$

$$\frac{1}{(4+t^2)(1+4t^2)}=\frac{A(1+4t^2)+B(4+t^2)}{(4+t^2)(1+4t^2)}$$

$$1=A(1+4t^2)+B(4+t^2)$$

Consider, $t^2=m$

$$1=A(1+4m)+B(4+m)$$

Put, $m=-4$

$$1=A(1-16)$$

$$1=A(-15)$$

$$A=\frac{-1}{15}$$

Put, $m=\frac{-1}{4}$

$$1=B\left(4-\frac{1}{4}\right)$$

$$1=B\times\frac{15}{4}$$

$$B=\frac{4}{15}$$

Put A and B in (i),

$$\int\left(\frac{-1}{15(4+t^2)}+\frac{4}{15\times 4(\frac{1}{4}+t^2)}\right)dt$$

$$\frac{-1}{15}\times\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+\frac{1}{15}\times\frac{1}{\frac{1}{2}}\tan^{-1}(2t)+C$$

$$\frac{-1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right)+\frac{2}{15}\times\tan^{-1}(2\sin\theta)+C$$

20.

Let $I = \int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ ....(i)

$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \sin x \cos x}{\sin^4 x + \cos^4 x} dx \quad \left(\text{Applying } \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right)$..(ii)

Adding, we get

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}$$

Divide numerator & denominator by $\cos^4 x$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\tan x . \sec^2 x}{\tan^4 x + 1} dx$$

Put $\tan^2 x = t$

$2\tan x . \sec^2 x\, dx = dt$

$x = 0, t = 0$

$x = \frac{\pi}{2}, t = \infty$

$$2I = \frac{\pi}{2} \times \frac{1}{2}$$

$$I = \frac{\pi}{8} \tan^{-1} x \Big]_0^{\infty}$$

$$= \frac{\pi}{8}\left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{16}$$

21. $xdy - ydx = \sqrt{x^2 + y^2}dx$

put y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

we get

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{x^2 + v^2x^2} + vx}{x}$$

$$\Rightarrow x\frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

intergrating

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log c$$

$$\Rightarrow \left|v + \sqrt{1 + v^2}\right| = |cx|$$

$$\Rightarrow \left|\frac{y}{x} + \sqrt{1 + \frac{y}{x^2}^2}\right| = |cx|$$

$$\Rightarrow \left\{y + \sqrt{x^2 + y^2}\right\}^2 = c^2x^4$$

as required

OR

We have

$$2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$$

$$\frac{dx}{dy}=\frac{2xe^{\frac{x}{y}}-y}{2ye^{\frac{x}{y}}}$$

sub $x=vy$

$$\frac{dx}{dy}=v+y\frac{dv}{dy}$$

so,

$$\Rightarrow v+y\frac{dv}{dy}=\frac{2ve^{v}-1}{2e^{v}}$$

$$\Rightarrow y\frac{dv}{dy}=-\frac{1}{2e^{v}}$$

$$\Rightarrow 2ye^{v}dv=-dy$$

$$\Rightarrow 2\int e^{v}dv=-\int\frac{1}{y}dv$$

$$\Rightarrow 2e^{v}=-\log|y|+\log c$$

$$\Rightarrow 2e^{v}=-\log\left|\frac{y}{c}\right|$$

$$\Rightarrow 2e^{\frac{x}{y}}=\log\left|\frac{c}{y}\right|$$

as required

22. If given lines are $\vec{r}_1=\hat{i}+\hat{j}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}_2=2\hat{i}+\hat{j}-\hat{k}+\mu\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$

Comparing with $\vec{r}=\vec{a}_1+\lambda\vec{b}_1,\ \vec{r}=\vec{a}_2+\mu\vec{b}_2$

$\vec{a}_1=\hat{i}+\hat{j},\quad \vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2=2\hat{i}+\hat{j}-\hat{k},\quad \vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$

$\Rightarrow \vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}=3\hat{i}-\hat{j}-7\hat{k}$$

$|\vec{b}_1\times\vec{b}_2|=\sqrt{9+1+49}=\sqrt{59}$

Shortest distance

$$=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2-\vec{a}_1\right)}{\vec{b}_1\times\vec{b}_2}\right|=\left|\frac{(3\hat{i}-\hat{j}-7k).\left(\hat{i}-k\right)}{\vec{b}_1\times\vec{b}_2}\right|$$

$$=\frac{10}{\sqrt{59}}$$

23. We know that equation of plane passing through 3 points.

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$$

$$\Rightarrow\begin{vmatrix}x-3 & y+1 & z-2\\ 2 & 3 & 2\\ -4 & 0 & 4\end{vmatrix}=0$$

$\Rightarrow\ x-3\ \ 12-0\ -\ y+1\ \ 8+8\ +\ z-2\ \ 0+12\ =0$

$\Rightarrow 12x-36-16y-16+12z-24=0$

$\Rightarrow 12x-16y+12z-76=0$

$\Rightarrow 3x-4y+3z-19=0$

Also, perpendicular distance of $P(6,5,9)$ to the plane $3x-4y+3z-19=0$

$$=\frac{|3\times6-4\times5+3\times9-19|}{\sqrt{9+16+9}}$$

$$=\frac{6}{\sqrt{34}}\text{ units}$$

SECTION – D

24. Let $A = IA$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

so,

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

$R_3 \to \frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

$R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

$R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

Hence,

$$A^{1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

OR

We have $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ and $f(x) = x^3 - 23x - 40$

$\therefore$ $f(A) = A^3 - 23A - 40I$

Now,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$$

$$A.A = A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^2.A = A^3 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$$A.A = A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$$

$$A^2.A = A^3 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$$

$f(A) = A^3 - 23A - 40I$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - 23\begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Hence, A is the root of the polynomials $f(x) = x^3 - 23x - 40$.

25. $f(x) = (x-2)^4(x+1)^3$

$$f'(x) = 3(x-2)^4(x+1)^2 + 4(x+1)^3(x-2)^3$$
$$= (x-2)^3(x+1)^2[3(x-2)+4(x+1)]$$
$$= (x-2)^3(x+1)^2[3x-6+4x+4]$$
$$= (x-2)^3(x+1)^2[7x-2]$$

$$f'(x) = 0 \Rightarrow (x-2)^3(x+1)^2[7x-2] \Rightarrow x = -1, \frac{2}{7}, 2$$

Let us examine the behaviour of $f'(x)$, slightly to the left and right of each of these three values of x

(i)

$x = -1$:

When $x < -1$; $f'(x) > 0$

When $x > -1$; $f'(x) > 0$

$\Rightarrow x = -1$ is neither a point of local maxima nor minima

$\Rightarrow$ It may be a point of inflexion

(ii)

$x = \frac{2}{7}$

When $x < \frac{2}{7}$; $f'(x) > 0$

When $x > \frac{2}{7}$; $f'(x) < 0$

$\Rightarrow x = \frac{2}{7}$ is a point of local maxima

$$f\left(\frac{2}{7}\right) = \left(\frac{2}{7} - 2\right)^4\left(\frac{2}{7}+1\right)^3 = \left(\frac{-12}{7}\right)^4\left(\frac{9}{7}\right)^3 = \frac{2^8 \times 3^{10}}{7^7}$$

$\Rightarrow$ The local maximum value is $\frac{2^8 \times 3^{10}}{7^7}$

(iii)

$x = 2$

When $x < 2$; $f'(x) < 0$

When $x > 2$; $f'(x) > 0$

$\Rightarrow x = 2$ is a point of local minima

$f(2) = (2-2)^4(2+1)^3 = 0$

$\Rightarrow$ The local minimum value is 0

26. Curve1 is circle, $x^2 + y^2 = 4$, vertex = (0,0), Radius = 2

Curve2 is parabola, $y^2 = 3(2x - 1)$, vertex $= \left(\frac{1}{2}, 0\right)$

On solving the two equations, we get

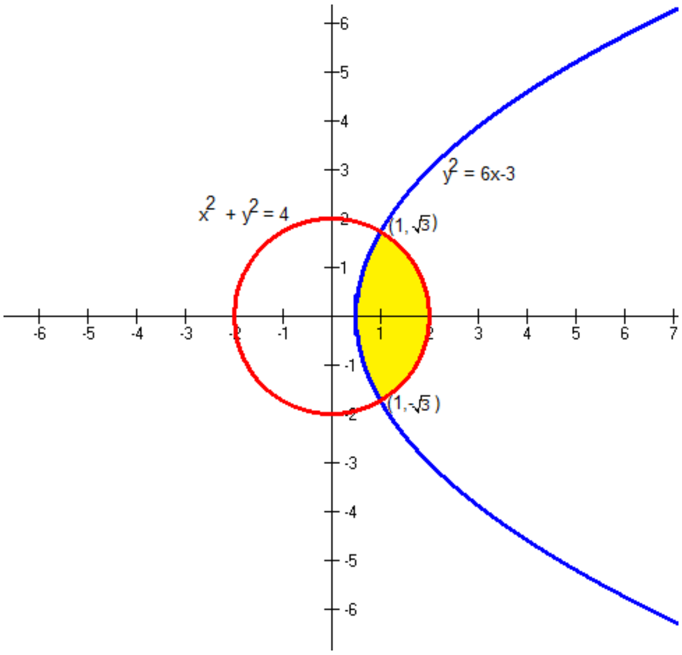
$x^2 + 3(2x - 1) = 4$

$\Rightarrow (x + 7)(x - 1) = 0$

$\Rightarrow x = 1, -7$

$x = -7$ is not possible since y^2 must be positive

Hence, x= 1



The region is symmetric about the x-axis. The region above the x-axis, bounded by the parallel lines x = 1/2, x = 1 and x = 1, x = 2.

$$\text{Required Area} = 2\left\{\int_{1/2}^{1} y_{C_2}\,dx + \int_{1}^{2} y_{C_1}\,dx\right\}$$

$$= 2\left\{\sqrt{3}\int_{1/2}^{1}\sqrt{2x-1}\,dx + \int_{1}^{2}\sqrt{4-x^2}\,dx\right\}$$

$$= 2\left\{\left[\sqrt{3}.\frac{2}{3}.\frac{(2x-1)^{3/2}}{2}\right]_{1/2}^{1} + \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_{1}^{2}\right\}$$

$$= \left\{\left[\frac{1}{\sqrt{3}}(2x-1)^{3/2}\right]_{1/2}^{1} + \left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_{1}^{2}\right\}$$

$$= 2\left[\frac{1}{\sqrt{3}} + 2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\frac{1}{2}\right]$$

$$= 2\left[\pi - \frac{\pi}{3} + \frac{2-3}{2\sqrt{3}}\right] = 2\left[-\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}\right]$$

$$= \left[-\frac{1}{\sqrt{3}} + \frac{4\pi}{3}\right] \text{sq. units.}$$

OR

Given : $y = \frac{5}{2}x - 5; x + y - 9 = 0; y = \frac{3}{4}x - \frac{3}{2}$

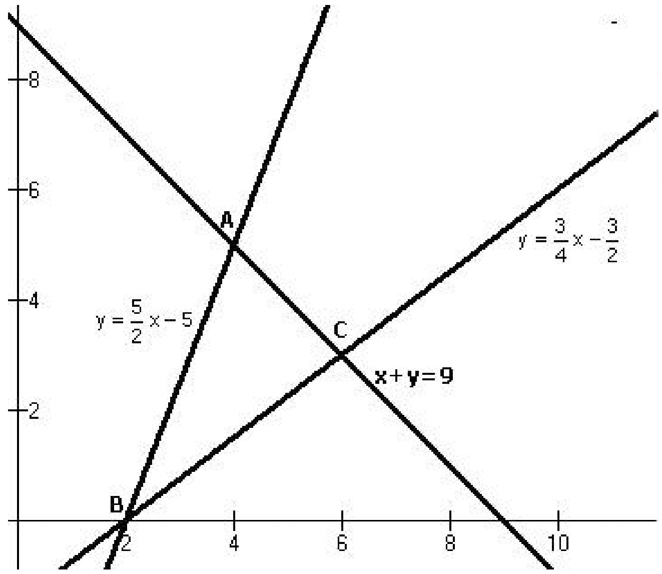
Point of intersection of the pair of lines $y = \frac{5}{2}x - 5; x + y - 9 = 0$ is (4,5)

Point of intersection of the pair of lines $y = \frac{5}{2}x - 5; y = \frac{3}{4}x - \frac{3}{2}$ is (2,0)

Point of intersection of the pair of lines $y = \frac{3}{4}x - \frac{3}{2}; x + y - 9 = 0$ is (6,3)

The area bounded by the 3 lines is the area of the triangle formed by the 3 lines.

Let ΔABC have vertices A(2,0) B(4,5) and C (6,3)



Area (△ABC) = area under segment AB + area under segment BC

- area under segment AC

$$\int_2^4\left(\frac{5}{2}x-5\right)dx+\int_4^6(-x+9)dx-\int_2^6\left(\frac{3}{4}x-\frac{3}{2}\right)dx$$

$$=\left[\frac{5x^2}{4}-5x\right]_2^4+\left[\frac{-x^2}{2}+9x\right]_4^6-\left[\frac{3x^2}{8}-\frac{3x}{2}\right]_2^6$$

$$=\left(\frac{5\times4^2}{4}-5\times4\right)-\left(\frac{5\times2^2}{4}-5\times2\right)+\left(\frac{-6^2}{2}+9\times6\right)-\left(\frac{-4^2}{2}+9\times4\right)-\left(\frac{3\times6^2}{8}-\frac{3\times6}{2}\right)+\left(\frac{3\times2^2}{8}-\frac{3\times2}{2}\right)$$

$$=20-20-(5-10)+(-18+54)-(-8+36)-\left(\frac{27}{2}-9\right)+\left(\frac{3}{2}-3\right)$$

$$=5+36-28-\frac{9}{2}-\frac{3}{2}=7 \text{ square units}$$

27. Equation of the plane passing through the intersection

of the planes x + y + z = 1 and 2x + 3y + 4z = 5 is :

$$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$$

$$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-(1+5\lambda)=0$$

This plane has to be perpendicular to the plane x - y + z = 0.

Thus,

$$(1+2\lambda)1+(1+3\lambda)(-1)+(1+4\lambda)1=0$$

$$1+2\lambda-1-3\lambda+1+4\lambda=0$$

$$1+3\lambda=0$$

$$\lambda=-\frac{1}{3}$$

Thus, the equation of the plane is :

$$\left(1+2\left(-\frac{1}{3}\right)\right)x+\left(1+3\left(-\frac{1}{3}\right)\right)y+\left(1+4\left(-\frac{1}{3}\right)\right)z-\left(1+5\left(-\frac{1}{3}\right)\right)=0$$

$$\left(1-\frac{2}{3}\right)x+(1-1)y+\left(1-\frac{4}{3}\right)z-\left(1-\frac{5}{3}\right)=0$$

$$\frac{x}{3}-\frac{z}{3}+\frac{2}{3}=0$$

$$x-z=-2$$

Thus, the distance of this plane form the origin is :

$$\left|\frac{-(-2)}{\sqrt{1^2+0^2+1^2}}\right|=\left|\frac{2}{\sqrt{2}}\right|=\sqrt{2}$$

OR

Any point in the line is

$2+3\lambda, -4+4\lambda, 2+2\lambda$

The vector equation of the plane is given as

$\vec{r}\cdot\left(\hat{i}-2\hat{j}+k\right)=0$

Thus the cartesian equation of the plane is $x-2y+z=0$

Since the point lies in the plane

$(2+3\lambda)1+(-4+4\lambda)(-2)+(2+2\lambda)1=0$

$\Rightarrow 2+8+2+3\lambda-8\lambda+2\lambda=0$

$\Rightarrow 12-3\lambda=0$

$\Rightarrow 12=3\lambda$

$\Rightarrow \lambda=4$

Thus, the point of intersection of the line and the plane is: $2+3\times 4, -4+4\times 4, 2+2\times 4$

$\Rightarrow 14,12,10$

Distance between $(2, 12, 5)$ and $(14, 12, 10)$ is:

$d=\sqrt{(14-2)^2+(12-12)^2+(10-5)^2}$

$\Rightarrow d=\sqrt{144+25}$

$\Rightarrow d=\sqrt{169}$

$\Rightarrow d=13$ units

28. Let x hectares of land be allocated to crop X and y hectares to crop Y.

Total profit = Rs. $(10500x + 9000y)$

Linear programming problem is,

Max. $Z = 10500x + 9000y$

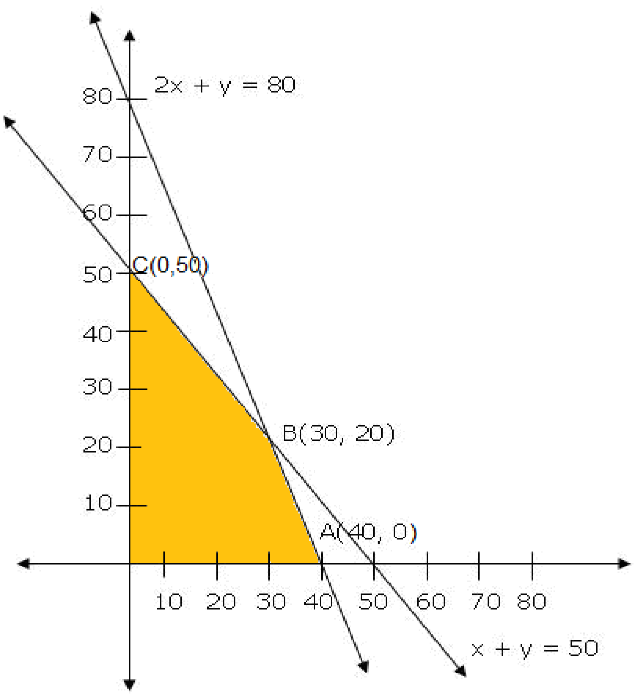
s.t $x + y \le 50$ (constraint related to land)

$20x + 10y \le 800$ (constraint related to use of hectare)

$2x + y \le 80$

$x \ge 0, y \ge 0$

Graphically the problem can be represented as



Graph

Corner point	z = 10500x + 9000y
0(0, 0)	0
A(40, 0)	420000
B(30, 20)	495000 → Maximize
C(0, 50)	450000

Hence, society will get the maximum profit of Rs. 4,95,000 by allocating 30 hectares for crop X and 20 hectares for crop Y.

29. N = 2

Success = 'throwing a doublet with a pair of dice'.

p = P (throwing a doublet with a pair of dice)

$$p=\frac{6}{36}=\frac{1}{6}$$

$$q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

$$P(X=0)={}^{2}C_{0}p^{0}q^{2}=\left(\frac{5}{6}\right)^{2}=\frac{25}{36}$$

$$P(X=1)={}^{2}C_{1}p^{1}q^{1}=2\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)=\frac{10}{36}$$

$$P(X=2)={}^{2}C_{2}p^{2}q^{0}=\left(\frac{1}{6}\right)^{2}=\frac{1}{36}$$

Hence the distribution of X is:

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$
XP(X)	0	$\frac{10}{36}$	$\frac{2}{36}$
$X^2P(X)$	0	$\frac{10}{36}$	$\frac{4}{36}$

$$\mu=\sum_{i=1}^{n} p_i x_i$$

$$\therefore \mu=\frac{12}{36}=\frac{1}{3}=0.33$$

$$\sigma^2=\sum p_i(x_i-\mu)^2$$

$$=\sum p_i x_i^2-\mu^2$$

$$=\frac{14}{36}-\frac{1}{9}=\frac{7}{18}-\frac{1}{9}=\frac{5}{18}=0.28$$